Exercise 217

The velocity V (in centimeters per second) of blood in an artery at a distance x cm from the center of the artery can be modeled by the function $V = f(x) = 500(0.04 - x^2)$ for $0 \leq x \leq 0.02$.

a. Find $x = f^{-1}(V)$

b. Interpret what the inverse function is used for.

c. Find the distance from the center of an artery with a velocity of 15 cm/sec, 10 cm/sec, and 5 cm/sec.

Solution

Solve the given function for x.

$$V = 500(0.04 - x^2)$$

Divide both sides by 500.

$$\frac{V}{500} = 0.04 - x^2$$

Subtract 0.04 from both sides.

$$\frac{V}{500} - 0.04 = -x^2$$

Multiply both sides by -1.

$$0.04 - \frac{V}{500} = x^2$$

Take the square root of both sides.

$$\pm\sqrt{0.04 - \frac{V}{500}} = x$$

Therefore, choosing the positive sign since $0 \leq x \leq 0.2$,

$$x = f^{-1}(V) = \sqrt{0.04 - \frac{V}{500}}.$$

The inverse function is used when you have a velocity, and you want to know the distance from the center of an artery.

$$V = 15\ \frac{\text{cm}}{\text{s}} \qquad \Rightarrow \qquad x = \sqrt{0.04 - \frac{15}{500}} = 0.1 \text{ cm}$$

$$V = 10\ \frac{\text{cm}}{\text{s}} \qquad \Rightarrow \qquad x = \sqrt{0.04 - \frac{10}{500}} \approx 0.141 \text{ cm}$$

$$V = 5\ \frac{\text{cm}}{\text{s}} \qquad \Rightarrow \qquad x = \sqrt{0.04 - \frac{5}{500}} \approx 0.173 \text{ cm}$$